

# 日々の計算ドリル

## ～数学Ⅲ微積分編～

《目次》  
P 1～P 7 微分編  
P 9～P 19 積分編

夏までに必ずできるようになっておきたい計算問題達です。

ただ、できるようになればよいわけではなく、  
スイスイ楽勝でできるようになってもらいたいです  
だから**毎日15分間だけでよいのでやりましょう！**

がんばってくださいね

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## 1 基本型

次の関数を微分せよ。

$$(1) \ y = x^{-\frac{5}{2}}$$

$$y' = -\frac{5}{2}x^{-\frac{7}{2}}$$

$$(2) \ y = \frac{1}{\sqrt[3]{x}}$$

$$y' = -\frac{1}{3}x^{-\frac{4}{3}} \quad \left( = -\frac{1}{3x\sqrt[3]{x}} \right)$$

$$(3) \ y = \sin x + x$$

$$y' = \cos x + 1$$

$$(4) \ y = \sin x - \tan x$$

$$y' = \cos x - \frac{1}{\cos^2 x}$$

$$(5) \ y = 3^x$$

$$y' = 3^x \log 3$$

$$(6) \ y = \log|x-1|$$

$$y' = \frac{1}{x-1}$$

$$(7) \ y = (x^2 - x - 2)(x^2 + x + 2)$$

$$y' = 4x^3 - 2x - 4$$

$$(8) \ y = x^2 \sin x$$

$$y' = x(2 \sin x + x \cos x)$$

$$(9) \ y = (3x + 1)e^x$$

$$y' = (3x + 4)e^x$$

$$(10) \ y = x \log x$$

$$y' = \log x + 1$$

$$(11) \ y = \frac{x+1}{x^2+1}$$

$$y' = \frac{-x^2 - 2x + 1}{(x^2 + 1)^2}$$

$$(12) \ y = \frac{2x+1}{x^2-2x-1}$$

$$y' = \frac{-2x(x+1)}{(x^2-2x-1)^2}$$

## 2 合成型(○<sup>α</sup>のタイプ)

次の関数を微分せよ。

$$(1) y = (2x^3 + 3)^6$$

$$y' = 36x^2(2x^3 + 3)^5$$

$$(2) y = (2x + 3)^{\frac{1}{3}}$$

$$y' = \frac{2}{3}(2x + 3)^{-\frac{2}{3}} \left( = \frac{2}{3\sqrt[3]{(2x + 3)^2}} \right)$$

$$(3) y = \sqrt{x^2 + 1}$$

$$y' = x(x^2 + 1)^{-\frac{1}{2}} \left( = \frac{x}{\sqrt{x^2 + 1}} \right)$$

$$(4) y = \frac{1}{\sqrt[3]{3x + 4}}$$

$$y' = -(3x + 4)^{-\frac{4}{3}} \left( -\frac{1}{(3x + 4)\sqrt[3]{3x + 4}} \right)$$

$$(5) y = \sin^4 x$$

$$y' = 4\sin^3 x \cos x$$

$$(6) y = \cos^3 x$$

$$y' = -3\cos^2 x \sin x$$

$$(7) y = \tan^5 x$$

$$y' = \frac{5\tan^4 x}{\cos^2 x}$$

$$(8) y = (2e^x + 3)^4$$

$$y' = 8e^x(2e^x + 3)^3$$

$$(9) y = (\log x)^2$$

$$y' = \frac{2\log x}{x}$$

$$(10) y = \sqrt{x + \sqrt{x}}$$

$$y' = \frac{2\sqrt{x} + 1}{4\sqrt{x}\sqrt{x + \sqrt{x}}}$$

### 3 合成型( $\sin \circlearrowleft$ , $\cos \circlearrowleft$ , $\tan \circlearrowleft$ のタイプ)

次の関数を微分せよ。

$$(1) y = \sin 3x \quad y' = 3 \cos 3x$$

$$(2) y = \cos(2x + 5) \quad y' = -2 \sin(2x + 5)$$

$$(3) y = \tan \frac{x}{3} \quad y' = \frac{1}{3 \cos^2 \frac{x}{3}}$$

$$(4) y = \sin(2x^2 - 3x + 1) \quad y' = (4x - 3) \cos(2x^2 - 3x + 1)$$

$$(5) y = \cos(\tan x) \quad y' = -\frac{\sin(\tan x)}{\cos^2 x}$$

$$(6) y = \tan(\log x) \quad y' = \frac{1}{x \cos^2(\log x)}$$

$$(7) y = \sin(xe^x) \quad y' = (1 + x)e^x \cos(xe^x)$$

$$(8) y = 4 \cos\left(\frac{x+1}{x^2+1}\right) \quad y' = \frac{4(x^2+2x-1)}{(x^2+1)^2} \sin\left(\frac{x+1}{x^2+1}\right)$$

#### 4 合成型( $e^\circ$ , $a^\circ$ のタイプ)

次の関数を微分せよ。

$$(1) y = e^{-2x} \quad y' = -2e^{-2x}$$

$$(2) y = e^{\frac{1}{3}x+2} \quad y' = \frac{1}{3}e^{\frac{1}{3}x+2}$$

$$(3) y = e^{\sin x} \quad y' = (\cos x)e^{\sin x}$$

$$(4) y = e^{x^2+x+1} \quad y' = (2x+1)e^{x^2+x+1}$$

$$(5) y = e^{x \cos x} \quad y' = (\cos x - x \sin x)e^{x \cos x}$$

$$(6) y = 3^{4x+5} \quad y' = 4 \cdot 3^{4x+5} \cdot \log 3$$

$$(7) y = 5^{x^3} \quad y' = 3x^2 \cdot 5^{x^3} \cdot \log 5$$

**5 合成型**( $\log |\bigcirc|$ ,  $\log_a |\bigcirc|$ のタイプ)

次の関数を微分せよ.

(1)  $y = \log |3x + 1|$

$$y' = \frac{3}{3x + 1}$$

(2)  $y = \log |2 - x|$

$$y' = -\frac{1}{2 - x}$$

(3)  $y = \log |x^2 - x - 3|$

$$y' = \frac{2x - 1}{x^2 - x - 3}$$

(4)  $y = \log |\sin x|$

$$y' = \frac{\cos x}{\sin x} \left( = \frac{1}{\tan x} \right)$$

(5)  $y = \log \frac{x}{2x + 1}$

$$y' = \frac{1}{x} - \frac{2}{2x + 1}$$

(6)  $y = \log \left( \tan \frac{x}{2} \right)$

$$y' = \frac{1}{\sin x}$$

(7)  $y = \log \left( \tan x + \frac{1}{\cos x} \right)$

$$y' = \frac{1}{\cos x}$$

(8)  $y = \log_2 |x - 3|$

$$y' = \frac{1}{(x - 3)\log 2}$$

(9)  $y = \log_3 (x^2 + 1)$

$$y' = \frac{2x}{(x^2 + 1)\log 3}$$

## 6 対数微分法

次の関数を微分せよ。

$$(1) y = x^x \quad (x > 0)$$

$$y' = x^x (\log x + 1)$$

$$(2) y = x^{\sin x} \quad (x > 0)$$

$$y' = x^{\sin x} \left\{ (\cos x) \log x + \frac{\sin x}{x} \right\}$$

$$(3) y = \sqrt[3]{\frac{x(x+2)^2}{x-1}}$$

$$y' = \sqrt[3]{\frac{x(x+2)^2}{x-1}} \cdot \frac{(x-2)(2x+1)}{3x(x+2)(x-1)}$$

$$(4) y = \frac{\sqrt[3]{x-1}}{x(x+2)}$$

$$y' = \frac{(-5x^2 + 2x + 6)\sqrt[3]{x-1}}{3x^2(x-1)(x+2)^2}$$

$$(5) y = (\log x)^x$$

$$y' = (\log x)^x \left\{ \log(\log x) + \frac{1}{\log x} \right\}$$

## 7 総合編

次の関数を微分せよ。

$$(1) y = (x^2 - 1)\sqrt{3 - 2x}$$

$$y' = \frac{-5x^2 + 6x + 1}{\sqrt{3 - 2x}}$$

$$(2) y = \cos^2 2x$$

$$y' = -4 \sin 2x \cos 2x \quad (= -2 \sin 4x)$$

$$(3) y = \sqrt{1 - \cos 2x}$$

$$y' = \frac{\sin 2x}{\sqrt{1 - \cos 2x}}$$

$$(4) y = \sin 2x \cos^3 x$$

$$y' = 10 \cos^5 x - 8 \cos^3 x$$

$$(5) y = e^{\sin(x^2+1)}$$

$$y' = 2xe^{\sin(x^2+1)} \cos(x^2 + 1)$$

$$(6) y = 2^{\tan x}$$

$$y' = \frac{2^{\tan x}}{\cos^2 x} \log 2$$

$$(7) y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$y' = \frac{4}{(e^x + e^{-x})^2}$$

$$(8) y = \log |\log x|$$

$$y' = \frac{1}{x \log x}$$

$$(9) y = \log \left\{ \cos \left( 2x - \frac{\pi}{4} \right) \right\}$$

$$y' = -2 \tan \left( 2x - \frac{\pi}{4} \right)$$

$$(10) y = \log \sqrt{\frac{1 + \sin x}{1 - \sin x}}$$

$$y' = \frac{1}{\cos x}$$

## 1 基本型

次の不定積分を計算せよ。

$$(1) \int x^4 dx \quad \frac{1}{5}x^5 + C$$

$$(2) \int x^{\frac{5}{3}} dx \quad \frac{3}{8}x^{\frac{8}{3}} + C$$

$$(3) \int \frac{dx}{x^2} \quad -\frac{1}{x} + C$$

$$(4) \int \frac{dx}{\sqrt{x}} \quad 2\sqrt{x} + C$$

$$(5) \int \frac{2}{x} dx \quad 2\log|x| + C$$

$$(6) \int \sqrt{x}(x+4)dx \quad \frac{2}{5}x^{\frac{5}{2}} + \frac{8}{3}x^{\frac{3}{2}} + C$$

$$(7) \int \frac{x-1}{x} dx \quad x - \log|x| + C$$

$$(8) \int \frac{2x^4 - 5x^3 + 3x^2 - 4}{x^3} dx \quad x^2 - 5x + 3\log|x| + \frac{2}{x^2} + C$$

$$(9) \int (\cos x - 1)dx \quad \sin x - x + C$$

$$(10) \int (2\sin x + 3)dx \quad -2\cos x + 3x + C$$

$$(11) \int \frac{4}{\cos^2 x} dx \quad 4\tan x + C$$

$$(12) \int (\sin x + \cos x)dx \quad -\cos x + \sin x + C$$

$$(13) \int \frac{\sin^2 x}{1-\cos x} dx \quad x + \sin x + C$$

$$(14) \int (e^x - 2) dx = e^x - 2x + C$$

$$(15) \int (2e^x + 5) dx = 2e^x + 5x + C$$

$$(16) \int 2^x dx = \frac{2^x}{\log 2} + C$$

$$(17) \int 10^x dx = \frac{10^x}{\log 10} + C$$

## 2 合成関数の微分の逆型

$$\textcircled{1} \int \bigcirc^{\alpha} \times \bigcirc' dx = \frac{1}{\alpha+1} \bigcirc^{\alpha+1} + C \text{ 型}$$

次の不定積分を求めよ。

$$(1) \int (2x-1)^4 dx = \frac{1}{10} (2x-1)^5 + C$$

$$(2) \int \frac{dx}{(3x-2)^2} = -\frac{1}{3(3x-2)} + C$$

$$(3) \int \sqrt{5x+1} dx = \frac{2}{15} (5x+1)^{\frac{3}{2}} + C$$

$$(4) \int \frac{dx}{\sqrt{x+3}} = 2\sqrt{x+3} + C$$

$$(5) \int \sqrt[3]{2x+3} dx = \frac{3}{8} (2x+3)^{\frac{4}{3}} + C$$

$$(6) \int x^2 (x^3 + 1)^2 dx = \frac{1}{9} (x^3 + 1)^3 + C$$

$$(7) \int x \sqrt{x^2 + 4} dx = \frac{1}{3} (x^2 + 4)^{\frac{3}{2}} + C$$

$$(8) \int (3x^2 + 1) \sqrt[3]{x^3 + x} dx = \frac{3}{4} (x^3 + x)^{\frac{4}{3}} + C$$

$$(9) \int \sin^3 x \cos x dx = \frac{1}{4} \sin^4 x + C$$

$$(10) \int \cos^5 x \sin x dx = -\frac{1}{6} \cos^6 x + C$$

$$(11) \int \frac{\tan^8 x}{\cos^2 x} dx = \frac{1}{9} \tan^9 x + C$$

$$(12) \int (1 - \cos^2 x) \sin x dx = -\cos x + \frac{1}{3} \cos^3 x + C$$

$$(13) \int \frac{(\log x)^4}{x} dx = \frac{1}{5}(\log x)^5 + C$$

$$(14) \int \frac{\log x}{x} dx = \frac{1}{2}(\log x)^2 + C$$

$$(15) \int \frac{dx}{x \log x} = \log|\log x| + C$$

$$(16) \int e^x (e^x + 1)^8 dx = \frac{1}{9}(e^x + 1)^9 + C$$

$$(17) \int \frac{(\sqrt{x} + 1)^3}{\sqrt{x}} dx = \frac{1}{2}(\sqrt{x} + 1)^4 + C$$

$$(18) \int \frac{2x - 5}{(x^2 - 5x - 4)^3} dx = -\frac{1}{2(x^2 - 5x - 4)^2} + C$$

$$(19) \int \frac{\cos x}{\sin^2 x} dx = -\frac{1}{\sin x} + C$$

$$(20) \int \frac{x}{2+x^2} \log(2+x^2) dx = \frac{1}{4} \left\{ \log(2+x^2) \right\}^2 + C$$

$$(21) \int \frac{1+\tan x}{\cos^2 x} dx = \frac{1}{2}(1+\tan x)^2 + C$$

$$\textcircled{2} \quad \int \frac{\bigcirc'}{\bigcirc} dx = \log |\bigcirc| + C \quad \text{型}$$

次の不定積分を求めよ。

$$(1) \int \frac{2x+3}{x^2+3x+5} dx \quad \log(x^2+3x+5) + C$$

$$(2) \int \frac{x}{x^2+2} dx \quad \frac{1}{2} \log(x^2+2) + C$$

$$(3) \int \frac{e^x}{e^x+1} dx \quad \log(e^x+1) + C$$

$$(4) \int \frac{dx}{\tan x} \quad \log |\sin x| + C$$

$$(5) \int \tan x \, dx \quad -\log |\cos x| + C$$

$$(6) \int \frac{1}{e^x+1} dx \quad -\log(e^{-x}+1) + C$$

$$(7) \int \frac{1}{e^{-x}+1} dx \quad \log(e^x+1) + C$$

※ (6), (7)のヒント→分子・分母に  $e^{-x}$ ,  $e^x$  を掛けてみると…

$$\textcircled{3} \quad \begin{cases} \int \sin \bigcirc \times \bigcirc' dx = -\cos \bigcirc + C \\ \int \cos \bigcirc \times \bigcirc' dx = \sin \bigcirc + C \\ \int \frac{1}{\cos^2 \bigcirc} \times \bigcirc' dx = \tan \bigcirc + C \end{cases}$$

次の不定積分を求めよ。

$$(1) \int e^x \cos(e^x + 1) dx \quad \sin(e^x + 1) + C$$

$$(2) \int x \sin x^2 dx \quad -\frac{1}{2} \cos x^2 + C$$

$$(3) \int \frac{\sin(\log x)}{x} dx \quad -\cos(\log x) + C$$

$$(4) \int \frac{\cos(\log x)}{x} dx \quad \sin(\log x) + C$$

$$(5) \int \frac{\cos x}{\cos^2(\sin x)} dx \quad \tan(\sin x) + C$$

$$(6) \int \frac{1}{x \cos^2(\log x)} dx \quad \tan(\log x) + C$$

$$(7) \int \sin x \cos(\cos x) dx \quad -\sin(\cos x) + C$$

$$\textcircled{4} \quad \begin{cases} \int e^\circ \times \bigcirc' dx = e^\circ + C \\ \int a^\circ \times \bigcirc' dx = \frac{a^\circ}{\log a} + C \end{cases} \quad \text{型}$$

次の不定積分を求めよ。

$$(1) \int e^{5x} dx \quad \frac{1}{5} e^{5x} + C$$

$$(2) \int (2x+1)e^{x^2+x} dx \quad e^{x^2+x} + C$$

$$(3) \int x e^{x^2} dx \quad \frac{1}{2} e^{x^2} + C$$

$$(4) \int e^{\sin x} \cos x \, dx \quad e^{\sin x} + C$$

$$(5) \int \frac{e^{\tan x}}{\cos^2 x} dx \quad e^{\tan x} + C$$

$$(6) \int 2^{5x} dx \quad \frac{2^{5x}}{5 \log 2} + C$$

$$(7) \int \frac{5^{\log x}}{x} dx \quad \frac{5^{\log x}}{\log 5} + C$$

### 3 三角関数のいろいろな積分

次の不定積分を求めよ。

$$(1) \int \cos^2 x \, dx \quad \frac{1}{2}x + \frac{1}{4}\sin 2x + C$$

$$(2) \int \sin^2 3x \, dx \quad \frac{1}{2}x - \frac{1}{12}\sin 6x + C$$

$$(3) \int \sin 5x \cos 3x \, dx \quad -\frac{1}{16}\cos 8x - \frac{1}{4}\cos 2x + C$$

$$(4) \int \cos 5x \cos 2x \, dx \quad \frac{1}{14}\sin 7x + \frac{1}{6}\sin 3x + C$$

$$(5) \int \sin^3 x \, dx \quad -\cos x + \frac{1}{3}\cos^3 x + C$$

$$(6) \int \sin^5 x \, dx \quad -\cos x + \frac{2}{3}\cos^3 x - \frac{1}{5}\cos^5 x + C$$

$$(7) \int \cos^3 x \, dx \quad \sin x - \frac{1}{3}\sin^3 x + C$$

$$(8) \int \cos^3 x \sin^2 x \, dx \quad \frac{1}{3}\sin^3 x - \frac{1}{5}\sin^5 x + C$$

## 4 分数関数の積分

次の不定積分を求めよ。

$$(1) \int \frac{dx}{2x-1} \quad \frac{1}{2} \log |2x-1| + C$$

$$(2) \int \frac{dx}{x^2 - 3x + 2} \quad \log \left| \frac{x-2}{x-1} \right| + C$$

$$(3) \int \frac{dx}{4x^2 - 1} \quad \frac{1}{4} \log \left| \frac{2x-1}{2x+1} \right| + C$$

$$(4) \int \frac{2x^2 - x + 3}{2x+1} dx \quad \frac{1}{2} x^2 - x + 2 \log |2x+1| + C$$

$$(5) \int \frac{x^3 - 2x^2 - 5}{x^2 - 2x - 3} dx \quad \frac{1}{2} x^2 + 2 \log |x+1| + \log |x-3| + C$$

$$(6) \int \frac{x^4}{x^2 - 1} dx \quad \frac{1}{3} x^3 + x + \frac{1}{2} \log \left| \frac{x-1}{x+1} \right| + C$$

## 5 部分積分

次の不定積分・定積分を求めよ。

$$(1) \int x \log x \, dx \quad \frac{1}{2}x^2 \log x - \frac{1}{4}x^2 + C$$

$$(2) \int x^2 \sin x \, dx \quad -x^2 \cos x + 2x \sin x + 2 \cos x + C$$

$$(3) \int e^x \cos x \, dx \quad \frac{1}{2}e^x(\sin x + \cos x) + C$$

$$(4) \int_0^\pi e^x \sin x \, dx \quad \frac{1}{2}(e^\pi + 1)$$

$$(5) \int_1^e x^2 \log x \, dx \quad \frac{1}{9}(2e^3 + 1)$$

$$(6) \int_1^e (\log x)^2 \, dx \quad e - 2$$

$$(7) \int_0^{\sqrt{e}} x \log(x^2 + 1) \, dx \quad \frac{1}{2}\{(e+1)\log(e+1) - e\}$$

$$(8) \int_1^e \frac{\log x}{x^3} \, dx \quad \frac{1}{4}\left(1 - \frac{3}{e^2}\right)$$

## 6 置換積分

次の定積分を求めよ。

$$(1) \int_0^{\sqrt{2}} \sqrt{2-x^2} dx \quad \frac{\pi}{2}$$

$$(2) \int_0^{\frac{3}{2}} \frac{dx}{\sqrt{3-x^2}} \quad \frac{\pi}{3}$$

$$(3) \int_0^1 \frac{dx}{x^2+1} \quad \frac{\pi}{4}$$

$$(4) \int_0^{\sqrt{2}} \frac{dx}{x^2+2} \quad \frac{\sqrt{2}\pi}{8}$$

$$(5) \int_0^1 \frac{x+1}{(x^2+1)^2} dx \quad \frac{\pi}{8} + \frac{1}{2}$$

$$(6) \int_{-2}^{-\frac{3}{2}} \frac{dx}{\sqrt{1-(x+2)^2}} \quad \frac{\pi}{6}$$

$$(7) \int_0^{\frac{\pi}{6}} \frac{dx}{\cos x} \quad \frac{1}{2} \log 3$$

$$(8) \int_1^4 e^{-\sqrt{x}} dx \quad \frac{4e-6}{e^2}$$